① $a\begin{bmatrix}3\\2\\1\end{bmatrix}$, $b=\begin{bmatrix}6\\5\\4\end{bmatrix}$ $c=\begin{bmatrix}-1\\1\\4\end{bmatrix}$ (2)

nearest to C

$c-a=\begin{bmatrix}-4\\-1\\0\end{bmatrix}$ $\|c-a\|=\begin{bmatrix}-4\\-1\\0\end{bmatrix}[4\ -1\ 0]=\sqrt{16+1}=\sqrt{15}=3.87$

$c-b=\begin{bmatrix}-7\\-4\\-3\end{bmatrix}$ $\|c-b\|=\begin{bmatrix}-7\\-4\\-3\end{bmatrix}[-7\ -4\ -3]=\sqrt{49+16+9}=8.60$

Vector a is nearest vector to C

$a=\begin{bmatrix}2\\4\\0\end{bmatrix}$ $b=\begin{bmatrix}1\\2\\-1\end{bmatrix}$ $\text{angle}<(a,b)=\cos\left(\frac{a^Tb}{\|a\|\|b\|}\right)$

$a^Tb=[2\ 4\ 0]\begin{bmatrix}1\\2\\-1\end{bmatrix}=2+8=\boxed{10}$ (2)

$\|a\|=\begin{bmatrix}2\\4\\0\end{bmatrix}[2\ 4\ 0]=\sqrt{4+16}=\sqrt{20}$

$\|b\|=\begin{bmatrix}1\\2\\-1\end{bmatrix}[1\ 2\ -1]=\sqrt{1+4+1}=\sqrt{6}$

$\text{angel}=\cos^{-1}\left[\frac{10}{10.95}\right]$

$\rightarrow \cos^{-1}\rightarrow 24.04$

Q2

(4)

$$X^T = \begin{bmatrix} 1 & 12 & 5 & 15 & 10 \end{bmatrix}$$

$$y = \begin{bmatrix} 155 & 100 & 130 & 85 & 115 \end{bmatrix}$$

$\rightarrow$ correlation $= \dfrac{\tilde{a}\,\tilde{b}}{\|\tilde{b}\|\|\tilde{a}\|}$ $\quad$ $\text{mean}^x = 8.6$ $\quad$ $\text{mean}^y = 117$

$$\tilde{a}^T = \begin{bmatrix} -7.6 & 3.4 & -3.6 & 6.4 & 1.4 \end{bmatrix}$$

$$\|\tilde{a}\| = [\tilde{a}^T \tilde{a}] = \sqrt{-7.6^2 + 3.4^2 + 7.6^2 \dots} = \boxed{11.489}$$

$$\tilde{b} = \begin{bmatrix} 38 & -17 & 13 & -32 & -2 \end{bmatrix}$$

$$\|\tilde{b}\| = \sqrt{38^2 + 17^2 + 13^2 + 32^2 + 2^2} = \boxed{54.129}$$

(4)

$$\tilde{a}.\tilde{b} = \begin{bmatrix} -7.6 \\ 3.4 \\ -3.6 \\ 6.4 \\ 1.4 \end{bmatrix} \begin{bmatrix} 38 & -17 & 13 & -32 & -2 \end{bmatrix} = -601$$

$$\text{Correlation} = \frac{-601}{11.48 * 54.129} = -0.99$$

ارتباط عكسي قوي

(2) find α β α to make vectors a, b

$$a = \alpha \begin{bmatrix} 3 \\ 4 \end{bmatrix} \qquad b = \beta \begin{bmatrix} 4 \\ x \end{bmatrix}$$

$$b^T b = \begin{bmatrix} B4 \\ Bx \end{bmatrix} \begin{bmatrix} B4 & Bx \end{bmatrix}$$

$$16B^2 + B^2x^2 = 1$$

$$4B + Bx = 1$$

(1)

① $a^T \cdot b = 0$

② $a^T a = 1 \rightarrow \begin{bmatrix} 3\alpha \\ 4\alpha \end{bmatrix} \begin{bmatrix} 3\alpha & 4\alpha \end{bmatrix} = 1$

(3) Normalize $\frac{1}{STD}.(x - avg)$ (1)

$$a = \begin{bmatrix} 100 \\ 150 \\ 250 \\ 300 \end{bmatrix}$$

mean $a = 200$

$$std \sqrt{\frac{(x-ave)^2}{n}} = \boxed{79.056}$$

$$\tilde{a} = \begin{bmatrix} -100 \\ -50 \\ 50 \\ 100 \end{bmatrix} \rightarrow Normalize = \begin{bmatrix} -1.26 \\ -0.63 \\ 0.63 \\ 1.26 \end{bmatrix}$$

Normalize را چک کنید

$\frac{-1.26 - 1.26 + 0.63 - 0.63}{4} \rightarrow Mean = 0$ (2)

~~STD = 1~~

$$STD \simeq 1 \quad \frac{\sqrt{(-1.26)^2 + (1.26)^2 + (-0.63)^2 + (0.63)^2}}{\sqrt{4}} = 0.99 \simeq 1$$

$$\begin{bmatrix} 1 & 2 & -1 & 4 & 6 \\ \frac{1}{2} & 1 & -7 & 2 & -4 \\ 0 & 0 & 10 & -2 & 14 \\ 0 & 0 & 5 & -1 & 5 \end{bmatrix}$$

$row_4 = row_4 - row_3$

$$\begin{array}{l} [0\ 1\ -2\ 1\ 1] \\ [0\ 1\ -7\ 2\ -4]- \\ \hline [0\ 0\ 5\ -1\ 5] \end{array}$$

$\times_{-2}$

$$\begin{array}{l} 0\ 0\ -10\ +2\ -10 \\ 0\ 0\ 10\ -2\ 14\ + \\ \hline 0\ 0\ 0\ 0\ 4 \end{array}$$

$+row_3 \rightarrow$

$\rightarrow$ row4

no solution

$$\begin{bmatrix} 1 & 2 & -1 & 4 & 6 \\ 0 & 1 & -7 & 2 & -4 \\ 0 & 0 & 10 & -2 & 14 \\ 0 & 0 & 5 & -1 & 5 \end{bmatrix}$$

$row_4 = row_4 - row_2$

$$\begin{array}{r} [0\ 1\ -2\ 1\ 1] \\ [0\ 1\ -7\ 2\ -4]- \\ \hline 0\ 0\ 5\ -1\ 5 \end{array}$$

$\times_{-2}$

$$\begin{array}{r} 0\ 0\ -10\ +2\ -10 \\ 0\ 0\ 10\ -2\ 14+ \\ \hline 0\ 0\ 0\ 0\ 4 \end{array}$$

$+row_3 \rightarrow$ row4

no solution